Q1a

$a = e^{\ln a}$

a) $\left(\frac{3}{5}\right)^x = e^{\ln\left(\frac{3}{5}\right)^x}$

Rewrite power (x) as a coefficient

$= e^{x\ln\left(\frac{3}{5}\right)}$

$= \boxed{e^{-0.511x}}$

Q1b

b) $\left(\frac{4}{7}\right)^{3t} = e^{\ln\left(\frac{4}{7}\right)^{3t}}$ $\qquad a = e^{\ln a}$

$= e^{3t\ln\left(\frac{4}{7}\right)}$

$= \boxed{e^{-1.68t}}$

exponential decay since $k < 0$

or since $0 < \frac{4}{7} < 1$

Q2a

a) $0.7^{x+1} = e^{\ln 0.7^{x+1}}$ $\qquad a = e^{\ln a}$

rewrite power $(x+1)$ as a coefficient

$= e^{(x+1)\ln 0.7}$

$= e^{\underline{x\ln 0.7} + \underline{\ln 0.7}}$

rewrite using laws of powers

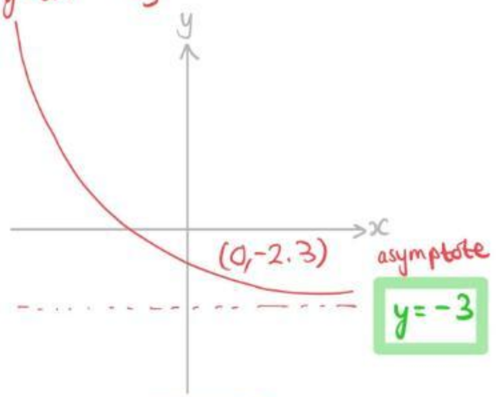
$= e^{x\ln 0.7} e^{\ln 0.7}$

$= 0.7e^{(\ln 0.7)x}$

$= \boxed{0.7e^{-0.357x}}$

Q2b

b) $y = 0.7^{x+1} - 3$



$y = 0.7e^{-0.357x} - 3$

$-0.357 < 0$
$\therefore$ exponential decay

shift 3 down in y-axis
(shift asymptote too!)

compress in y axis.

sub $x = 0$
$y = 0.7e^{-0.357(0)} - 3 = 0.7 - 3 = -2.3$

$\therefore$ y-axis intercept: $\boxed{(0, -2.3)}$

Q3a

a) $\ln x = \ln 7e^{-0.2t}$

rewrite as 2 terms

$= \ln 7 + \underbrace{\ln e^{-0.2t}}$

$$\boxed{\ln x = \ln 7 \quad -0.2t}$$

Q3b

b) since the final form contains e, raise both sides to the power of e

$e^{\ln y} = e^{4.1x + \ln 8}$

$y = e^{4.1x} e^{\ln 8}$

$$\boxed{y = 8e^{4.1x}}$$

Q4a

a) $\log y = \log 2x^{3/4}$

rewrite as two terms

$= \underline{\log 2} + \log x^{3/4}$

rewrite power as coefficient

$$\boxed{\log y = \log 2 + 0.75 \log x}$$

Q4b

b) Raise 10 to the power of either side of the eqn

$$10^{\log y} = 10^{4.7\log x + \log 12}$$

$$= 10^{4.7\log x}\,10^{\log 12}$$

$$y = 12\left(10^{4.7\log x}\right)$$

$$= 12\left(10^{\log x}\right)^{4.7}$$

rewrite using laws of powers

$$\boxed{y = 12x^{4.7}}$$

Q5a

a) $\log_2 y = \log_2(0.1 \times 2^{0.01x})$

rewrite as 2 terms

$$= \log_2 0.1 + \log_2 2^{0.01x}$$

$$= \log_2 10^{-1} + 0.01x$$

rewrite power (-1) as coefficient

$$\boxed{\log_2 y = -\log_2 10 + 0.01x}$$

Q5b

b) Raise both sides as powers of 3 (since it's the base)

$$3^{\log_3 y} = 3^{6.3x + \log_3 4}$$

rewrite using laws of powers

$$y = 3^{6.3x}\, 3^{\log_3 4}$$

$$\boxed{y = 4 \times 3^{6.3x}}$$

Q6a

a) Sub $t=0$ into model

$$D = 20e^{0.1(0)} = \boxed{20 \text{ deer}}$$

Q6b

b) Sub in $D = 2 \times 20 = 40$

$$40 = 20e^{0.1t}$$

$$2 = e^{0.1t}$$

$$\ln 2 = \ln e^{0.1t}$$

$$\ln 2 = 0.1t$$

$$t = \frac{1}{0.1}\ln 2 = 10\ln 2 = \boxed{6.93y}$$

Q6c

c) $\boxed{\text{model suggests population increases indefinitely, but population has a limit (space, resources, food etc).}}$

Q6d

d) Do we seperate at $t = 25y$ or when $D > 400$, t?

choose lowest t

sub $D = 400$

$$400 = 20e^{0.1t}$$

$$20 = e^{0.1t}$$

$$\ln 20 = 0.1t$$

$$t = 10\ln 20 = 29.957y$$

$$29.957 > 25y$$

seperate at $\boxed{25y}$

Q7a

a) $A = 10$ $t = 4$

$$10 = 5e^{k(4)}$$

$$2 = e^{4k}$$

$$\ln 2 = 4k$$

$$k = \frac{1}{4}\ln 2 = \boxed{0.173} \quad (3sf)$$

Q7b

b) Read initial A from model or sub $t=0$: $A = 5e^{k0} = 5$

Now sub in $A = 3\times 5 = 15$

$$15 = 5e^{kt}$$

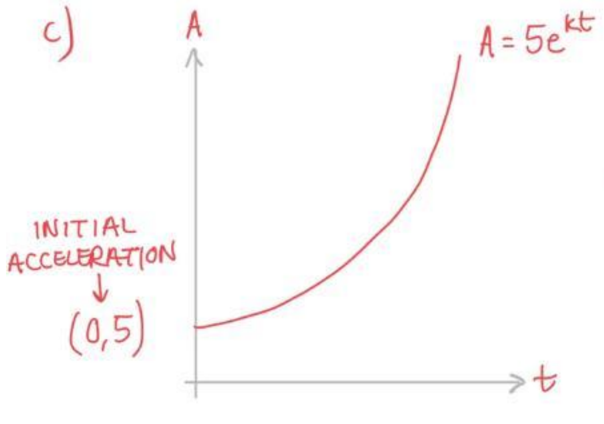
$$3 = e^{kt}$$

$$\ln 3 = kt$$

$$t = \frac{1}{k}\ln 3 = \boxed{6.34\,s}$$

Q7c

c)



$k>0$
$\therefore$ exponential growth

Read initial acceleration from equation or sub in $t=0$

$$A = 5e^{k(0)} = 5$$

Q8a

a) HALF LIFE $t = 5700$, $y_{5700} = \frac{1}{2} y_0$

$= \frac{1}{2}(100) = 50$

$50 = 100e^{-k5700}$

$\frac{1}{2} = e^{-5700k}$

$\ln\frac{1}{2} = -5700k$

$k = \frac{-\ln\frac{1}{2}}{5700} = \boxed{1.22 \times 10^{-4}}$ (3sf)

Q8b

b) sub $y = 0.5$

$0.5 = 100e^{-kt}$

$\frac{0.5}{100} = e^{-kt}$

$\ln\frac{0.5}{100} = -kt$

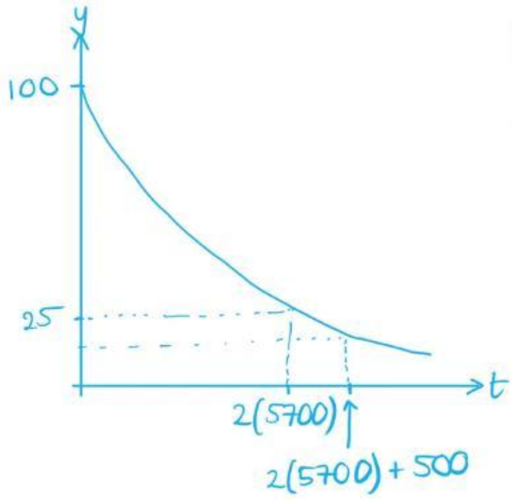
use of k

$t = -\frac{1}{k}\ln\frac{0.5}{100} = 43569.98y$

$= \boxed{43600y}$ (3sf)

Q8c

c)



25 is $\frac{1}{4}$ of 100

$\therefore$ this is equivalent to two half lives having past

$t = 2 \times 5700 + 500 = 11900$

$y = 100e^{-(1.22 \times 10^{-4})(11900)}$

$= \boxed{23.5g}$ (3sf)

Q9A

a)

t	0	2	4	6
$\log_5 N$	3.29	3.64	3.97	4.35

Q9B

b) $m = \dfrac{4.4-3.3}{6-0} = \dfrac{1.1}{6} = 0.183$

at y intercept $N = 200 \longrightarrow c$

$$\boxed{\log_5 N = 0.183t + \log_5 200}$$

Q9C

c) To get an equation where t is no longer in the power, we take $\log_a$ of both sides of the model.

$a^{x} = b$

$\log_a b = x$

$a \rightarrow \text{base} = 5$

Rewrite $N = N_0 a^{kt}$ in the form of the line of best fit. Then read off values of N_0 and k.

$$\log_5 N = \log_5 N_0 a^{kt}$$

$$= \log_5 N_0 + \log_5 a^{kt}$$

$$\log_5 N = \log_5 N_0 + kt(\log_5 5)^{1}$$

$$\log_5 N = \log_5 200 + 0.183t$$

$$\boxed{\begin{aligned} N_0 &= 200 \\ k &= 0.183 \\ a &= 5 \end{aligned}}$$

Q10A

a) y intercept $= \ln c = 1.10$

$c = e^{1.10} = 3.00$ (3sf)

gradient $= m = \frac{\Delta \ln D}{\Delta t} = \frac{0.262 - 1.10}{2 - 0} = -0.419$ (3sf)

Line of best fit:

$$\boxed{\ln D = -0.419t + \ln 3.00}$$

Q10B

b) Rewrite eqn ① in the form of eqn ② since it has no unknowns

$\ln D = \ln A e^{-kt}$

$= \ln A + \ln e^{-kt}$

① $\ln D = \ln A \quad -kt$

② $\ln D = \ln 3.00 - 0.419t$

compare ① and ② to read off the values of A & k

$$\therefore \boxed{\begin{aligned} A &= 3.00 \\ k &= 0.419 \end{aligned}}$$

Q10c

c) D= 1% of initial dose (A)

$0.01\cancel{A} = \cancel{A}e^{-kt}$

$\ln 0.01 = \ln e^{-kt}$

$= -kt$

$t = \frac{-1}{k} \ln 0.01 = \frac{-1}{0.419} \ln 0.01$ use exact value of k

$= 11.0\text{h}$

$\boxed{11\text{h } 0\text{m}}$

Q11A

a) yr 1 $3100 = P_1 (1)^k$

$\boxed{P_1 = 3100}$

yr 4 $6200 = (3100)(4)^k$

$2 = 4^k$

$\boxed{k = \frac{1}{2}}$

Q11b

b) $\log P = \log P_1 a^k$

$= \log P_1 + \log a^k$

$\log P = \log P_1 + k \log a$

$$\boxed{\log P = \log 3100 + \frac{1}{2} \log a}$$

Q11C

c) The model is based on data from the first four years. Twelve years is a long way away from the years used to create the model, this would require risky extrapolation.